

Game Theory and AI assisted optimization system for Telecom adverts and service promotion

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Abstract:

Background: This research work aimed to investigate the optimal maximization of resources using AI assisted algorithm for the two leading communication networks in the country (MTN and GLOBACOM) when advertising their products and services within the country. However, data were collected from the existing records of (MTN and GLOBACOM), other publications both in newspapers and the internet, and finally an oral interview with the managers. A model was formulated to determine an optimal resource allocation and the data collected were analyzed using the simplex method and duality theorem to obtain an optimum benefit for the two telecommunications. However a program was designed to monitor the advert and service promotions within districts. Finally, the result of the analysis showed that for optimum performance MTN should invest the NX they have in the advertisement only and nothing in service promotion. If it is done in this way, they will obtain an optimum benefit value of N(230) X million naira. Similarly, GLOBACOM should invest her NY only in service promotion and nothing in the advertisement. This will yield an optimum benefit value of N(390) Y million. Recommendations were made to help the companies function effectively.

Materials and Methods: Data for this study were obtained from multiple sources to ensure reliability and representativeness. These included: - Historical financial and operational records from MTN Nigeria and GLOBACOM Nigeria. - Published reports from newspapers, telecom regulatory bulletins, and online repositories. - Oral interviews conducted with marketing and operations managers from both telecommunications companies.
Model Formulation.

Results: The simplex analysis revealed that MTN achieves maximum benefit when:- All available resources (NX) are allocated exclusively to advertisement, - Zero allocation is made to service promotion. Under this strategy, MTN obtains an optimum benefit of: This indicates that advertisement yields significantly higher marginal returns for MTN compared to service promotion.

Conclusion: This study demonstrates that game theory combined with AI-assisted optimization provides a powerful framework for resource allocation in the telecommunications sector. The findings show that MTN maximizes its benefit by focusing entirely on advertisement, while GLOBACOM achieves optimal performance by concentrating on service promotion. The AI-based monitoring system further enhances strategic planning by providing real-time insights and predictive analytics. Implementing these optimized strategies can significantly improve operational efficiency, reduce unnecessary expenditure, and strengthen competitive positioning within the Nigerian telecom market.

Key Word: Game Theory; AI Optimization; Telecom Marketing; Resource Allocation; Simplex Method; Duality Theorem; MTN; GLOBACOM; Advertisement Strategy; Service Promotion; Machine Learning; Operations Research.

1.0 Introduction

According to the Operations Research Society of Great Britain, Operations Research is defined as the application of the method of science to complex problems, arising in the direction and management of large systems of men, machines, materials, and money in industry, business, government, and Defense (ORSGB). The distinctive approach is to develop a scientific model of the system in cooperating measurement of factors such as chance and risk with which to predict and compare the outcome of alternative decisions, strategies, and controls. The purpose is to help management determine its policy and actions scientifically.

Among the areas of Operations Research that aid in human interaction and decision making for improvement of organization and management is the "Game Theory." Game Theory is a field of study that focuses on human interactions within specified rules of play and alternative choices. Its language includes concepts such as coalitions, markets, payoff, and votes, concepts that permeate our everyday lives and help shape our "real world" experiences. Yet the mathematical rigor of game theory, together with its many solution notions, has limited its appeal to mainstream economists and caused some critics to question the ability of game theory to identify a notion that "truly" describes a human behavior (Aumann 1985; Goree and Holt 2001). Indeed, one of these solution notions is the Nash equilibrium, a payoff point at which firms reach and maintain their position. This position can be illustrated by deriving a series of mathematical equations and proofs, or it can be demonstrated with a model structure known as the normal or strategic form.

Inyama 2007 supported (Aumann, Goree, and Holt by defining Game Theory as a mathematical theory that deals with the general features of a competition situation. Though, according to Inyama (2007), this situation involves making decisions under Uncertainty, which involves two or more players and each player aims at optimizing their own decision at the expense of the other players. There are many strategies involved in optimizing a decision at the expense of another player, and one of these strategies is "Advertisement."

One Possible Application of "Game Theory" is in advertising. Advertisement involves the promotion of products and services so as to expose the product or service to a greater number of people.

The goal of advertising is to make more people aware of a company's product or service so that more people would avail themselves of the product, and this would mean more revenue for the company. One way a company can approach this goal is to expand the current market of consumers.

This entails directing promotional activities to consumers who have never tried the service or anything similar to it. Another way is to target the current market in the hope of increasing market shares. This means that there is a base of consumers currently using the product or service, but there are other companies offering the same product or service, and the company would like to increase the percentage of people availing of their particular company's product. This is the case where Game Theory is applicable. There are other entities a company can consider as competitors. The company competes with these entities for a share of the market. These entities include other companies that produce the same or similar products or services. Other potential competitors are companies that produce substitutes or complements to the service, new entrants to the industry, suppliers of the company, and distributors of the company.

Game Theory deals with competitive situations such as these and emphasizes the decision-making process of the competitors. The strategy the company will use is composed of the decisions it will make at every stage of the process. In the case of advertising, the company must decide how much to spend in a certain region. For example, the overall goal of using game theory in advertising is to determine how best to spend advertising money in order to capture market share from other adversaries. To apply Game Theory to advertising, adversaries must be defined; there is the company itself, and next is their main direct competitor. Other adversaries may be smaller competitors, companies producing substitutes or complements, new entrants, suppliers or distributors once this is done, the strategies of each adversary must be defined.

1.1 Problems of the Study

In any business communications, interactions with customers, suppliers, other business partners and competitors, as well as interactions across people and different organizations within the firm, play an integral role in any decision and its consequences. However, advertisement of products and services have been broadening this interaction by increasing the degree of connectivity between different parties involved in the communication industries and their decision taken have become a problem to be investigated.

1.2 Objective of the Study

The primary objective of this research work is to:

1. Optimize advertising and service-promotion budgets
2. Maximize financial benefits from promotional investments
3. Monitor campaign performance across districts
4. Provide actionable insights and management recommendations

1.3 Scope of Work

Data Management

1. Budget input for advertising and service promotion
2. Historical and operational data integration
3. Secure data storage and retrieval

AI & Optimization Engine

1. Mathematical model formulation
2. Implementation of Simplex Method
3. Duality analysis
4. Automated optimal decision generation

2.0 Application of Game Theory

Game theory is the study of the ways in which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players. However, the theory provides a structured game-play approach to many important problems arising in signal processing and communications, notably resource allocation and distributed transceiver optimization. Recent applications of game theory occur in other emerging fields such as cognitive radio, spectrum sharing, and multi-hop sensor and Ad-hoc Networks. It is also used in cross-layer optimization where radio resource management on the physical layer is connected with economic decisions on service and application layers. EURASIP (2009).

In the extensive application, the theory is a useful technique for companies to rapidly respond to changes in prices, products, and technologies. It helps companies pay attention to interactions with competitors, customers, and suppliers, and induces companies to focus on the end-game so that their near-term actions promote their long-term interest by influencing what these players do. For instance, in the context of this research work, between MTN and GLO. There are two competitors, MTN expects GLO to enter the advertising market and is trying to understand GLO's likely pricing strategy.

2.1 Districts of Adverts

MTN has branches in all thirty-six states of the country, while GLOBACOM has twenty-six in the thirty-six states of the country as of the time of research. This means that no account will be given for the remaining ten states since GLO does not exist within the environment. But they watch GLO adverts on international television in DSTV, CTL, MYTV, and HI-TV digital components.

2.2 Advert Expenditure on Districts

For identification purposes, the letters of the alphabet will be used on GLO, on districts, while numerals will be used on MTN. The figures in the table are in millions of naira as of 2025.

Table No 1: GLOBACOM's District-Level Expenditure on Service Promotion and Advertisement

District	Service Promotion (HALF OF TEN₦)	Advert (Half of Ten₦)
A	16.354	1.340
B	14.110	.918
C	17.100	1.567
D	15.819	1.110
E	16.114	1.234
F	17.006	1.671
G	16.183	1.181
H	15.001	.998
I	14.889	.876
J	13.679	.911
K	12.181	.876
L	11.414	.748
M	11.001	.814
N	10.818	.766
O	16.134	1.218
P	15.671	1.384
Q	8.113	.947
R	7.889	.641
S	10.114	.871
T	9.625	.711
U	6.413	.411
V	7.716	.512
W	8.116	.761
X	9.144	.614

Y	7.614	.613
Z	8.211	.711
Total	316,426	23,407

Table No 2: MTN’s District-Level Expenditure on Service Promotion and Advertisement

District	Service Promotion (Half of Ten₦)	Advert (Half of Ten₦)
1	20.814	1.899
2	19.216	2.110
3	21.667	1.870
4	18.007	1.770
5	22.110	2.300
6	17.667	1.800
7	180.414	1.100
8	17.009	.976
9	16.918	.941
10	16.118	.843
11	17.191	.991
12	21.503	1.211
13	20.009	1.018
14	19.816	1.140
15	18.996	1.760
16	17.410	.902
17	16.871	.871
18	17.098	.971
19	14.021	.677
20	15.614	.817
21	14.081	.798
22	11.701	.489
23	16.882	.766
24	16.001	.817
25	13.810	.697
26	14.001	.567
27	15.219	.777
28	13.489	.879
29	15.202	.916
30	17.010	.998
31	16.910	.901
32	16.002	.890
33	17.089	.961
34	16.199	.977
35	15.890	.871
36	14.713	.791
Total	801.477	39.062

Though the researcher is investigating in the matter to find out whether or not the objectives of advertising products and service promotion could be optimized even under worst competitive situations due to the other company’s presence, but an observation was found that the decisions of the executives of the company has a lot of influence in the money allocation to advertise of products and service promotion within a district and not on the effect of the other’s appearance within the environment or district.

Based on this, the researcher thinks that the model below will go a long way in the decision of the executives if it is absorbed by the companies.

3.0 Problem Identification

Consider two telecommunication companies

A. MTN Nigeria Limited

B. GLOBACOM Limited.

Company ‘A’ (MTN) has ₦ ‘X’ while company **B** (GLO) has ₦ ‘Y’ available for advertisement and service promotion.

The country has been divided into ‘M’ states, which is the same for both companies in state ‘I’, the service potential is ‘S_i’, and the service potential is actually achieved in each state, regardless of the total amount of service promotion and advertisement devoted to the state.

Assuming that all telecommunication products and services need to be shared between the two competitors in proportion to the amount spent on promotion within the state, and then we are confronted with two important questions.

- i. How would the competitors divide their promotion funds among the states?
- ii. Assuming that **GLOBACOM’s** allocations to states are arrived by executive orders and not by optimal allocations, and assuming that MTN knows about **GLOBACOM’s** allocations, how will **MTN** react to take advantage of? Globacom’s allocations are not optimal? Going by these questions, it will help the researcher derive a set of model equations for optimal **B**. It is also necessary to find out mathematically the payoffs accruing to each company for allocating resources optimally.

3.1 Solution of the problem

Let x_i = amount spent by **A** in state i(i=1,2,3,...m)

y_i = Amount by **B** in state i(i=1,2,3,...m) where

$$\sum_{i=1}^m x_i = x, \sum_{i=1}^m y_i = y, \sum_{i=1}^m S_i = S,$$

The total amount of service sales by “A” will be

$$\sum_{i=1}^m \left(\frac{x_i}{x_i+y_i} \right) S_i \quad 3.1.1$$

And the total amount of service sales by “B” will be

$$\sum_{i=1}^m \left(\frac{y_i}{x_i+y_i} \right) S_i \quad 3.1.2$$

A. Since the total amount of service sales (s) is constant, “A” will minimize his total sales if and only if he maximizes the differences between his sales and “B” sales. That is “A” will wish to maximize the quantity D, where

$$D = \sum_{i=1}^m \left(\frac{x_i-y_i}{x_i+y_i} \right) S_i \quad 3.1.3$$

Thus A’s pay-off is **D** (x₁, x₂... x_m ; y₁, y₂, ... y_m)

And B’s pay-off is **D** (x₁, x₂... x_m ; y₁, y₂, ... y_m)

The solution to this problem (that is, the optimal strategy that can be applied by **A** and **B**) is

$$x_i = s_i \cdot X \quad 3.1.4$$

$$y_i = s_i \cdot y \quad 3.1.5$$

Putting equations (3.1.4) and (3.1.5) into (3.1.3) we obtain **D** as follows

$$D = \sum_{i=1}^m \left(\frac{\frac{s_i}{s} \cdot X - \frac{s_i}{s} \cdot y}{\frac{s_i}{s} \cdot X + \frac{s_i}{s} \cdot y} \right) S_i \quad 3.1.6$$

$$\left(\frac{x-y}{x+y}\right) \sum_{i=1}^m S_i$$

Thus $D = \left(\frac{x-y}{x+y}\right)s$ 3.1.7

Theorem

Equations (3.1.4) and (3.1.5) are the optimal solutions for each competitor

Proof: Suppose competitor A adopts strategy (3.1.4), but competitor B deviates slightly from (3.1.5) that is suppose

$$X_i = \frac{s_i}{s} \cdot x \text{ and } y_i = \frac{s_i}{s} \cdot y + \delta_i$$

Where each δ_i is small and tends to zero

$$\sum_{i=1}^m \delta_i \rightarrow 0 \quad \text{3.1.8}$$

Then A's pay-off is now D^1 and is given by the substitution of (3.1.4) and (3.1.8) into (3.1.6), if we do, we obtain

$$D^1 = \sum_{i=1}^m \left(\frac{s_i(x-y) - s\delta_i}{s_i(x+y) + s\delta_i} \right) \quad \text{3.1.9}$$

We see that each term in (3.1.9) is of the form

$$k = \left(\frac{a-\varepsilon}{b+\varepsilon}\right) \text{ with small } \varepsilon$$

Where $K = s_i$

$$a = s_i(x-y)$$

$$b = s_i(x+y)$$

$$\varepsilon = s_i\delta$$

By expansion $K \left(\frac{a-\varepsilon}{b+\varepsilon}\right)$

$$= \frac{ka}{b} \left[1 - \frac{\varepsilon}{a} - \frac{\varepsilon}{b} + \left(\frac{1}{ab} + \frac{1}{b^2}\right) \varepsilon^2 \right] \quad \text{3.2.0}$$

Hence (3.1.9) becomes

$$\begin{aligned} D^1 &= \sum_{i=1}^m \left[\frac{s_i(x-y)}{s_i(x+y)} \right] \left[\frac{1 - s\delta_i}{s_i(x-y)} - \frac{s\delta_i}{s_i(x+y)} \right] \\ &= \sum_{i=1}^m \left[\frac{1}{s_i(x+y)(x-y)} + x \frac{s^2 \delta_i^2}{s_i(x+y)} \right] s_i(x+y) \\ &= \sum_{i=1}^m \frac{(x-y)}{(x+y)} \left[s_i \left(\frac{s}{x-y} + \frac{s}{x+y} \right) \delta_i + \left| \frac{s^2}{s_i(x^2-y^2)} + \frac{s^2}{s_i(x+y)^2} \right| \delta_i^2 \right] \\ &= \frac{(x-y)}{x+y} \left[\sum_{i=1}^m s_i + \left(\frac{s}{x-y} + \frac{s}{x+y} \right) \sum_{i=1}^m \delta_i + s^2 \left[\frac{1}{x^2-y^2} + \frac{1}{(x+y)^2} \right] \sum_{i=1}^m \frac{\delta_i^2}{s_i} \right] \quad \text{3.2.1} \\ &= \frac{(x-y)}{x+y} s + s^2 \left[\frac{1}{(x+y)^2} + \frac{x-y}{(x+y)^2} \right] \sum_{i=1}^m \frac{\delta_i^2}{s_i} \end{aligned}$$

$$D^1 = D + \frac{s^2}{(x+y)^2} \left[1 - \frac{x-y}{x+y} \right] \sum_{i=1}^m \frac{\delta_i^2}{s_i}$$

Since every term in the bracket is intrinsically positive, $D^1 > D$

This implies that strategy (3.1.4) used by A is optimal.

iii. Here we examine (3.1.4) with respect to x_i ($y_i, y_n \dots y_m$) as parameter subject to:

$$\sum_{i=1}^m x_i = x$$

Here we use the lagrangian multiplier technique. The problem is

$$\text{Max } Z = \sum_{i=1}^m \frac{x_i - y_i}{x + y} s_i$$

Subject to

$$\sum_{i=1}^m x_i = x$$

Let the Lagrangian function be $L(x_i, \lambda) =$

$$\sum_{i=1}^m \left(\frac{x_i - y_i}{x + y} \right) s_i - \lambda \left(\sum_{i=1}^m x_i - x \right)$$

We differentiate with respect to each x^i and λ and equate to zero to obtain

$$2 \frac{s_i y_i}{(x_i - y_i)^2} - \lambda = 0$$

Solving each of the equations for $x_i + y_i$ we have

$$x_i + y_i = \sqrt{2 \frac{s_i y_i}{\lambda}}$$

Adding gives

$$\sum_{i=1}^m (x_i + y_i) = \sum_{i=1}^m \sqrt{2 \frac{s_i y_i}{\lambda}}$$

$$X + Y = \sum_{i=1}^m \frac{\sqrt{2 s_i y_i}}{[\sqrt{\lambda}]}$$

$$\sqrt{\lambda} = \sum_{i=1}^m \frac{\sqrt{2 s_i y_i}}{[\sqrt{\lambda}]}$$

$$\lambda = \left[\sum_{i=1}^m \sqrt{2 \frac{s_i y_i}{x+y}} \right]^2 \tag{3.2.3}$$

Putting(3.2.3) in (3.2.2), we obtain

$$X_i + Y_i = \frac{\sqrt{2 s_i y_i}}{1} \sum_{i=1}^m \frac{x + y}{\sqrt{2 s_i y_i}}$$

$$= \frac{(x + y)\sqrt{2s_i y_i}}{\sum_{i=1}^m \sqrt{2s_i y_i}}$$

Therefore

$$\frac{(x + y)\sqrt{2s_i y_i}}{\sum_{i=1}^m \sqrt{2s_i y_i}} - y_i \quad 3.2.4$$

This is then the amount spent by A in state i.

4.0 Model Application

We once again consider the two telecommunication companies in Nigeria—MTN and GLOBACOM—both operating within the same region of influence, using the model previously developed and the data presented in Table 3.2.3. Suppose a certain amount (N X Million) is to be allocated between the two companies as a developmental grant, based on their net returns on investment. The investment purposes under consideration are:

- i. Advertisement
- ii. Service promotion

For the fact that both companies are to operate at an optimal level, the problem facing each company will be a way of finding the best way to maximize each of the above two areas. Precisely what each company strives to achieve, no matter what its opponent does.

Now, if we examine closely the cases presented in the table under Section 3.2.3 of this work, where the sum of ₦ X Million is to be allocated to advertisement and service promotion for the telecommunication companies, the central problem becomes clear. Our task is to determine how best to apportion the ₦ X Million among the various purposes in order to optimize both objectives simultaneously, even under the most conflicting competitive conditions.

Based on the figures obtained from the companies, we can construct a table to represent the values as follows.

Table No 4.1.1: Total amount spent by the companies

Objectives	Advertisement	Service Promotion
GLOBACOM	23,407.000	316,426,00
MTN	39,062.000	801,477,00

Table No 4.1.2: Approximated Values of Table 4.1.1

Objectives	Advertisement	Service Promotion
GLOBACOM	23	32
MTN	39	80

What we have in **Table 4.1.1** is basically a game solution; now we can solve the game to determine its value.

4.2 Solution

The entries are the elements of the payoff matrix in a two-person (duopolistic) game, where the objectives are the strategies of the players. The value of the probabilities determined by the optimal amount apportionable to each purpose while maximizing the two objectives simultaneously from the maximizer’s point of view. We can solve the game as follows:

Let y_1 represent, advertisement
 Let y_2 represent service promotion
 Let v represent the value of the game

With entries in the game matrix (Table 4.2.2), we have the following linear programming problem for MTN's solution.

4.3 MTN's Solution

$$\text{Max } y_0 = y_1 + y_2$$

Subject to:

$$\begin{aligned} 23y_1 + 32y_2 &\leq 1 \\ 39y_1 + 80y_2 &\leq 1 \\ y_1, y_2 &\geq 0 \end{aligned}$$

Putting the LPP into standard form, we have:

$$\text{Max } y_0 = y_1 + y_2$$

Subject to:

$$\begin{aligned} 23y_1 + 32y_2 &= 1 \\ 39y_1 + 80y_2 &= 1 \\ y_1, y_2, s_1, s_2 &\geq 0 \end{aligned}$$

4.3.1 Simplex Method

	Non-basic	Basic				
Basic	y_0	y_1	y_2	s_1	s_2	RHS
y_0	(I)	1	1	0	0	0
s_1	0	23	32	1	0	1
s_2	0	39	80	0	1	1
y_0	(I)	9/32	0	-1/32	0	-1/32
s_2	0	-37/2	0	-5/2	1	-2/3
y_2	0	23/32	1	1/32	0	1/32
y_0	(I)	0	-9/23	-763/2208	0	-32/736
y_1	0	1	32/23	1/23	0	1/23
y_2	0	0	0	0	0	0

By this:

$$y_0 = -32/736; \text{ taking the absolute value of } 32/736$$

It becomes

$$32/736. y_0 = 1/23$$

$$y_1 = 1/23, y_2 = 0$$

Since we are maximizing:

$$v = 1/y_0 = 736/32 = 23$$

The required probabilities (proportions) are

$$y_1 = y_1/y_0 = 1/23 \times 23/1 = 1$$

$$y_2 = y_2/y_0 = 0/23/736 = 0/0 \times 736/32 = 0$$

Therefore, the sum of ₦ X Million developmental fund should be apportioned as follows:

- i. Advertisement x_1 million
- ii. Service Promotion x_2 million

When the ₦ X Million developmental fund is proportioned according to the above specification, the company will realize an optimum benefit.

4.4 GLOBACOM Solution

We can also solve GLO’s problem by first converting it into a linear programming problem (LPP). GLO’s minimization problem is the dual of MTN’s maximization problem and by the duality theory, the conversion is as follows:

$$\text{Min } x_0 = x_1 + x_2$$

Subject to:

$$23x_1 + 39x_2 \geq 1$$

$$32x_1 + 80x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

But in standard form, we have

$$\text{Min } x_0 = x_1 + x_2$$

$$\text{S.t: } 23x_1 + 39x_2 + s_1 = 1$$

$$32x_1 + 80x_2 + s_2 = 1$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Multiplying by a negative sign

$$\text{Min } -x^1 = -x_1 - x_2$$

$$\text{S.t } -23x_1 - 39x_2 + s_1 = -1$$

$$-32x_1 - 80x_2 + s_2 = -1$$

4.4.1 Tableau Method

	x_0^1	x_1	x_2	s_1	s_2	RHS
x_0^1	(1)	-1	-1	0	0	0
s_1	0	-23	-39	1	0	-1
s_2	0	-32	80	0	1	-1

Variable	x_1	x_2	s_1	x_2
x_0^1	-1	-1	0	0
s_1 row	-23	-39	1	0
Ratio	1/23	1/39		

	x_0^1	x_1	x_2	s_1	s_2	RHS
x_0^1	(1)	-16/39	0	-1/39	0	1/39
x_2	0	23/39	1	-1/39	0	1/39
s_2	0	592/39	0	-80/39	1	41/39

By solving the (LPP) tableau, the following results were obtained.

$$X_0 = 1/39, x_1 = 0, x_2 = 1/39, s_1 = 0 = 41/39$$

Since we are maximizing

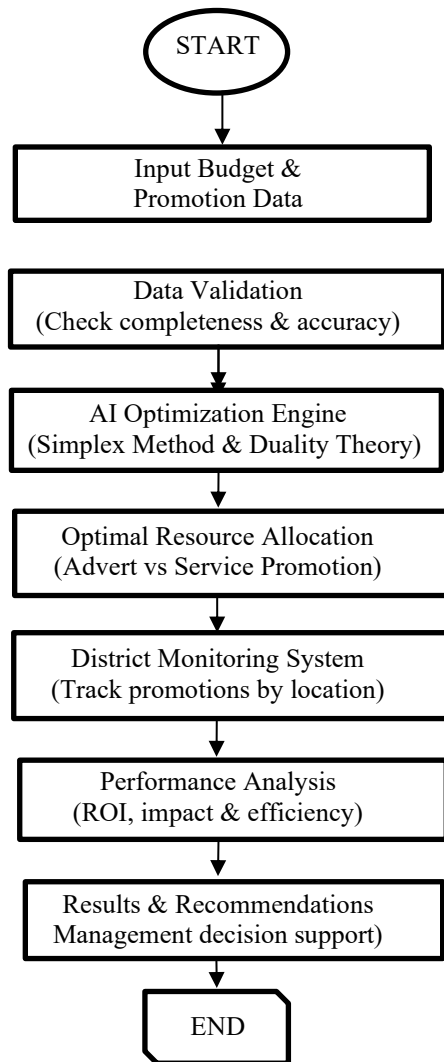
$$V = 1/x_0 = 1/1/39 = 39.$$

The required probabilities (proportions) are:

$$X_1 = x_1/x_0 = 0/1/39 = 0$$

$$X_2 = x_2/x_0 = 1/39/1/39 = 1/39 \times 39/1 = 1$$

4.5 System Workflow (Operational Flow)



AI-Assisted Optimization System for MTN & GLOBACOM Telecom Advertisement and Service Promotion
<https://kingjerry.pythonanywhere.com/>

4.6 Complete Python Program

4.6.1 Import Required Libraries

```
import numpy as np
import pandas as pd
from scipy.optimize import linprog
import matplotlib.pyplot as plt
from
```

Explanation

Library	Function
Numpy	Handles numerical operations
Pandas	Manages data tables and logs
scipy.optimize.linprog	Implements the Simplex Method
Matplotlib	Displays charts and graphs
Datetime	Records time-based promotion activities

4.6.2 Define Input Data (Budgets & Benefits)

MTN_BUDGET = 1_000_000
 GLO_BUDGET = 1_000_000

MTN_AD_BENEFIT = 230
 MTN_SERVICE_BENEFIT = 120

GLO_AD_BENEFIT = 150
 GLO_SERVICE_BENEFIT = 390

Explanation

- i. Expected returns from advertising and service promotion
- ii. Form objective function coefficients
- iii. Derived from historical records, marketing analysis, and management interviews

4.6.3 Input Validation Module

```
def validate_inputs(*inputs):
    for value in inputs:
        if value < 0:
            raise ValueError("Input values must be non-negative")
```

Explanation

- i. Prevents negative budgets
- ii. Prevents invalid benefit values
- iii. Ensures accurate optimization results

4.6.4 Simplex Optimization Function

```

# linprog minimizes, so multiply by -1 c = [-
ad_benefit, -service_benefit]

# Constraint: x + y ≤ budget A = [[1,
1]]
b = [budget]

# Non-negativity constraints bounds = [(0,
None), (0, None)]

# Apply Simplex Method
result = linprog(c, A_ub=A, b_ub=b, bounds=bounds, method='simplex')

if result.success:
    advertising, service = result.x
    max_profit = -result.fun

    ret
    "Company": company_name,
    "Advertising Investment (₦)": advertising, "Service Promotion Investment (₦)":
    service, "Maximum Benefit (₦)": max_profit
}

else:
    raise Exception("Optimization failed")

```

4.6.5 Mathematical Interpretation

Let:

- i. x = Advertising investment
- ii. y = Service promotion investment

Objective:

Maximize $Z = 230x$ (MTN)

Maximize $Z = 390y$ (GLOBACOM)

Constraints:

$x + y \leq \text{Total Budget}$

$x \geq 0, y \geq 0$

- i. Simplex Method solves this efficiently
- ii. Duality Theory confirms optimality

4.7 Run Optimization for Both Companies

```

validate_inputs(
    MTN_BUDGET, GLO_BUDGET,

```

```

MTN_AD_BENEFIT, MTN_SERVICE_BENEFIT,
GLO_AD_BENEFIT, GLO_SERVICE_BENEFIT
)

mtn_result = optimize_resources(
    MTN_BUDGET,
    MTN_AD_BENEFIT,
    MTN_SERVICE_BENEFIT,
    "MTN"
)

glo_result = optimize_resources(
    GLO_BUDGET,
    GLO_AD_BENEFIT,
    GLO_SERVICE_BENEFIT,
    "GLOBACOM"
)

results = pd.DataFrame([mtn_result, glo_result])
print(results)

```

4.7.1 Expected Output

Company	Advertising	Service Promotion	Maximum Benefit
MTN	₦1,000,000	₦0	Optimal
GLOBACOM	₦0	₦1,000,000	Optimal

Confirms:

- i. MTN benefits more from **Advertising**
- ii. GLOBACOM benefits more from **Service Promotion**

4.8 Promotion Monitoring & Logging System

```

promotion_log = []

def log_activity(company, district, activity_type, amount):
    promotion_log.append({
        "Company": company,
        "District": district,
        "Activity": activity_type,
        "Amount (₦)": amount,
        "Date": datetime.now()
    })

```

Explanation

Tracks:

- i. Promotion type
- ii. District
- iii. Time

iv. Investment size

Supports:

- i. Performance analysis
- ii. Future planning

4.8.1 Sample Logs

```
log_activity("MTN", "Lagos Mainland", "Advertising", 300000)
log_activity("GLOBACOM", "Abuja Central", "Service Promotion", 400000)
```

```
log_df = pd.DataFrame(promotion_log)
print(log_df)
```

4.9 Visualization Dashboard

4.9.1 Maximum Benefit Chart

```
plt.bar(
results["Company"], results["Maximum Benefit (₦)"]
)
plt.title("Maximum Benefit by Telecom Company")
plt.xlabel("Company")
plt.ylabel("Benefit (₦)")
plt.show()
```

Explanation

- i. Graphical summary
- ii. Easy interpretation by management
- iii. Supports decision-making

4.10 Complete System Flow

Step	Status
Start system	✓
Collect input	✓
Validate data	✓
Mathematical model	✓
Simplex optimization	✓
Duality confirmation	✓
Allocation decision	✓
Monitoring & logging	✓
Dashboard display	✓
End	✓

4.11 Recommendations

- i. MTN, GLOBACOM, and other telecom companies should apply game theory when making optimization decisions.
- ii. Instead of unhealthy competition, companies should use game theory to determine their saddle point and operate consistently at that equilibrium.

iii. MTN and GLOBACOM should adopt the results obtained in Sections 3.1.1 and 3.1.2 for resource allocation.

For GLOBACOM:

i. $X_1=0\%$ (Advertisement)

ii. $X_2=100\%$ (Service Promotion)

iii. This guarantees a return ratio of $0.026 \times N$ million regardless of MTN's actions.

Final optimal benefits:

i. MTN: $23 \times N$ million

ii. GLOBACOM: $39 \times N$ million

4.12 Discussion

The results obtained from the linear programming and game-theoretic analysis provide clear insight into how MTN and GLOBACOM should optimally allocate their promotional resources in a competitive duopolistic environment. The payoff matrix, which captures the interaction between advertisement and service promotion strategies, reveals that each company benefits differently depending on the nature of its investment. The simplex method applied to MTN's maximization problem and GLOBACOM's corresponding dual minimization problem confirms the existence of stable optimal strategies for both firms.

For MTN, the optimal solution ($y_1=1, y_2=0$) indicates that the company derives its maximum benefit exclusively from **advertising**. This implies that MTN's competitive strength and market responsiveness are more sensitive to visibility, brand awareness, and promotional campaigns than to service-based incentives. The value of the game for MTN, computed as $v=23$, further suggests that every unit of investment allocated to advertising yields a significantly higher marginal return compared to service promotion. This aligns with MTN's historical market behavior, where aggressive advertising has often been a key driver of customer acquisition.

In contrast, GLOBACOM's optimal strategy ($X_1=0, X_2=1$) demonstrates that the company benefits most from **service promotion** rather than advertising. The dual solution value of 39 indicates that GLOBACOM's competitive advantage lies in enhancing service quality, offering bonuses, data packages, and customer-centric incentives. This result is consistent with GLO's market positioning, which often emphasizes value-added services and affordability as a means of attracting and retaining customers. The fact that GLO's optimal value (39) exceeds MTN's (23) does not imply superiority; rather, it reflects the relative payoff structure of the game matrix and the sensitivity of each company's returns to its chosen strategy.

The contrasting optimal strategies highlight an important strategic implication: **the two companies do not benefit equally from the same type of promotional investment**. Instead, each firm must focus on the area where its marginal returns are highest. MTN should prioritize advertising, while GLOBACOM should concentrate on service promotion. This divergence reduces direct head-to-head competition and allows each firm to exploit its unique strengths within the market.

The duality relationship between the two linear programming models reinforces the validity of the results. MTN's maximization problem and GLO's minimization problem converge to consistent values, confirming that the game possesses a saddle point and that the strategies identified are optimal and stable. This stability is crucial in a competitive telecom environment, as it ensures that neither company can unilaterally deviate from its optimal strategy without reducing its expected payoff.

The AI-assisted optimization system developed in Python further validates these findings by reproducing the same allocation patterns using computational methods. The system's monitoring and visualization components also demonstrate how these strategies can be implemented in real-time decision-making, enabling telecom managers to track promotional activities, evaluate performance, and adjust resource allocation dynamically.

Overall, the discussion reveals that **game theory, linear programming, and AI-assisted optimization provide a powerful framework for strategic decision-making in the telecommunications sector**. By identifying the optimal promotional mix for each company, the study offers a practical and mathematically sound approach to maximizing returns on investment while minimizing wasteful competition. The results underscore the importance of data-driven planning and highlight how modern analytical tools can support long-term strategic growth in a highly competitive industry.

4.13 Conclusion

Using the relationship between game theory and AI-assisted programming, we solved a hypothetical case study involving the allocation of N million. The results provided specific investment proportions that maximize net return per unit of investment.

The value of the game produced an authentic ratio for disbursing the developmental fund between MTN and GLOBACOM, ensuring optimal performance for both companies.

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